An easy recipe for better double quantum dot qubits: just add electrons

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ICCMSE 2014
Overview

- What are Double quantum dots (DQDs)?
- How are double quantum dots used as qubits?
  - **Spin qubit:** 2 electrons, $|S\rangle$ and $|T\rangle$
  - *? Spin qubit:* $N$ electrons, $N$ even, $|S\rangle$ and $|T\rangle$ *?* (E.g. $N=6$)

Central Question of this talk:
Background: Double quantum dots

- Two 0-dimensional potential wells (quantum dots) placed next to each other
- Different types:
  - Vertical: wells in different heterostructure layers (in vertical z-direction)
  - Lateral: wells in same layer, spaced along x or y direction
2e DQD Spin Qubit
(another type of spin qubit)

\[ |1\rangle = \text{unpolarized triplet} \quad (S=1, S_z=0) \text{ state} \]
\[ |0\rangle = \text{singlet} \quad (S=0) \text{ state} \]
2-electron DQD Spin Qubits

\[ |1\rangle = \text{unpolarized triplet (S=1, } S_z=0\text{) state} \]
\[ |0\rangle = \text{singlet (S=0) state} \]

S(1,1)

T_0(1,1)

S(0,2)

\[ S(0,2) \]

\[ S(1,1) \]

\[ T_+(1,1) \]

\[ T_0(1,1) \]

Constant Magnetic Field \( B > 0 \)

\[ J = \text{“exchange energy”} \]

\[ g\mu_B B \]

B-field splits off polarized triplets \( T_+, T_- \)

Taylor et al., PRB 76, 035315
2e DQD Spin Qubit: Energies with $\Delta B$

$|1\rangle = \text{unpolarized triplet (S}=1, S_z=0\text{) state}$

$|0\rangle = \text{singlet (S}=0\text{) state}$

Magnetic Field $B > 0$ + small gradient $\Delta B > 0$

B-field splits off polarized triplets $T_+, T_-$

Moving: $S(0,2)$ to left adiabatically $\rightarrow A(\uparrow\uparrow)$

diabatically $\rightarrow S(1,1)$

Note: $A(s,-s)$ levels may be reversed from as shown: if $\Delta B$ is unknown we don’t know whether $A(\uparrow\downarrow)$ or $A(\downarrow\uparrow)$ is lower.

Taylor et al., PRB 76, 035315
2e DQD qubit: Manipulation

- **Z-rotation** (need H to split singlet and triplet states)
  - Exchange energy does this, so J determines Z-rotation speed:

\[ J = E_T - E_S \]

- **X-rotation** (need H to split ud and du states)
  - Magnetic field gradient does this, and comes about by:
    - Hyperfine interaction (different dots have different nuclear environments), so we could just wait in the low-J “park” region.
    - wire w/current
    - micromagnets
    - G-factor engineering (can give rise to an effective B-field gradient)

**Qubit Hamiltonian**

\[ H_{\text{eff}} = \begin{bmatrix} J(\ ) & B_z \\ B_z & 0 \end{bmatrix} = J \left( \frac{z+1}{2} \right) + (B_z)_x \]
Calculation for 2e Spin DQD qubit

Splitting between $|S\rangle$ and $|T\rangle$ = exchange energy $J$ = rate of qubit $z$-rotation

DQD potential (min of parabolas):

$E_0 = 14.78\text{meV}$, $L=20\text{nm}$, $B=1.7\text{T}$
Central questions of this talk

- Can we still make qubits with multiple electrons in each dot?
  - Do electrons form inert filled shells in dots?
- If they do make qubits, how do they compare with the 2e case?
- The case we consider:
  - $6e^-$ DQD
  - $2e^-$ DQD

* 2 e- fill lowest “shell” of dot (single valley assumption)

Are the **lowest lying states** of the 6-electron system a **singlet and triplet**, and is there a **gap** to higher states?
Method: Configuration Interaction

- Solve many-electron Hamiltonian

\[
\mathcal{H} = \sum_i \mathcal{H}_i + \sum_{i<j} \frac{e^2}{\kappa r_{ij}} \quad \mathcal{H}_i = \frac{(\vec{p} - e\vec{A})^2}{2m^*} + V(\vec{r}) + \frac{e}{m^*} \vec{S} \cdot \vec{B}
\]

- Basic procedure:

  - We use two independent implementations:

    - CI-1: 1e⁻ basis = \textbf{Fock Darwin}
      states at dot center
      \[
      \phi_{nm}^{\pm}(x,y) = \frac{1}{\ell_0} \sqrt{\frac{(n-|m|)!}{\pi \left(\frac{n+|m|}{2}\right)!}} \left(\frac{x \pm x_0 + i y \text{sgn} m}{\ell_0}\right)^{|m|} \\
      \times e^{-\frac{(x+x_0)^2+y^2}{2\ell_0^2}} \sum_{y_0} L_{\frac{|m|}{2}} \left(\frac{(x \pm x_0)^2 + y^2}{\ell_0^2}\right)
      \]

    - CI-2: 1e⁻ basis = \textbf{s-type Gaussians}
      at different centers
      \[
      g(x,y) = N e^{-\alpha_x (x-x_0)^2} e^{-\alpha_y (y-y_0)^2} \\
      \times e^{-i e B (y_0 x-x_0 y)}
      \]

- We use two independent implementations:
6e⁻ small-dot DQD (representative system)

- Singlet and unpolarized triplet isolated ground space
- Init & read-out: Regions with order meV splitting
- Manipulation: smooth avoided crossings

* 4 Charge Sectors *

\[ E_0 = 24.26 \text{meV}, L = 20 \text{nm}, B = 2.8 \text{T} \]
Comparison: $6e^-$ vs. $2e^-$

- Exchange energy (qubit z-rotation)

  - 6e has **two** plateaus

- Charge sensor sensitivity to the dot state

  - Same order of magnitudes – as nice as a 2e qubit
When does a 6 electron DQD form a good qubit?

Small Dots (~ 15, 25nm diameter in GaAs)

- Same prep, manip., and readout as 2e
- meV gaps between “qubit” and “non-qubit” states.

GOOD QUBIT

Large dots (~ 50nm diameter in GaAs)

- Gaps are smaller
- Intervening levels (e.g. multiple triplets below a singlet)
- Non singlet (or triplet) ground states?

COMPLICATED
Why the complications?

Factors at play: (Consider **single dot** properties)

**Dot Size**

Single dot exchange

**Magnetic field**

6-electrons in a single **large dot** \( (E_0 \sim 2\text{meV}) \)

**Design goals:** **small dots** \(<50\text{nm diameter}) \) and **variable magnetic field**
Motivation

Why not just use a 2-electron qubit?

- Multi-electron dots are easier to realize
- It’s free (no harder prep, manip, and measure)

And also:

- Multi-electron dots make the qubit better
  - more robust to disorder & noise  [ Barnes et al., PRB 84, 235309 (2011)
    Higginbotham et al, PRL 112, 026801 (2014) ]
  - More versatile due to larger number of charge sectors
Robustness to charge noise

FIG. 10. (Color online) Fractional change in exchange energy due to impurity vs impurity distance for different numbers of electrons using the energy-cutoff approximation with $E_c = 35, 26, 24, 17$ for $N = 2, 6, 10, 14$ electrons, respectively.

Barnes et al., PRB 84, 235309 (2011)
$J$-Plateaus: tunable with B-field

- Exchange energy **plateaus are tunable** by varying the magnetic field.
- **Two plateaus** could be useful for:
  - Separate **initialization** and **rotation plateaus** (want different $O$(magnitudes))
  - **Multiple speeds** of rotation = more possibilities for dynamical correction
So, can multi-electron DQD qubits be made?

Yes

**Theoretical support:**

3D Schrödinger-Poisson CAD simulations indicate that lithographic dots can be made small enough to display good multi-electron qubit behavior.
So, can multi-electron DQD qubits be made?  

YES

- Experimental support:

Higginbotham et al, PRL 112, 026801 (2014)

singlet-triplet coherence observed using many-electron DQD in GaAs
Applications of shell structure beyond DQDs

- Donor + Multi-electron Dot systems

- Donor Cluster systems
  - Donor Cluster + Donor Cluster
  - Donor Cluster + Dot
Conclusions

6e DQDs are theoretically viable as qubits:

- **Initialization, manipulation, and read-out** are similar to the 2e case, and are no harder to perform (perhaps easier)
- They are more robust to random charge impurities & charge noise (they screen better)
- They may offer richer control capability, depending on the tunable range of the DQD, perhaps resulting in increased robustness to control noise.
- In practice, current experimental dot are predicted to be on the ‘edge’ of the sizes needed to preserve their shell structure enough to make multi-electron qubits viable. Focus should be given to produce dots with **large confinement** energies and **tunable magnetic field**.

References for this talk:
EN, Edwin Barnes, J. P. Kestner, and S. Das Sarma, PRB 88, 195131
Barnes et al., PRB 84, 235309 (2011)
Acknowledgements & References

- References for this talk:
  - Barnes et al., PRB 84, 235309 (2011)

- The authors would like to acknowledge Professor Sankar Das Sarma for many helpful conversations and guidance in this work.